DISCRETE MATHEMATICS

COUNTING PROCEDURES

GRADES 11-12

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Executive Summary:
Throughout the unit students will be provided multiple entry points to working with counting procedures. All lessons are based off of group and/or partner work to build understanding. No mention will be made as to how students should approach the problems in each lesson so they could choose to draw a picture, make a table, notice a pattern, or any other method they choose. These multiple entry points allow students opportunities to make sense of the problems, reason abstractly and quantitatively, construct arguments and analyze reasoning of others, model mathematics, use appropriate tools, look for structure, and express general methods.

Teaching for understanding requires teachers to help students feel comfortable in sharing and discussing ideas, asking questions, and taking risks. With nearly no direction on possible strategies or solutions, this unit requires cognitive effort and perseverance through potential apprehension towards the unknown. When students are fully engaged in doing mathematics we are better able to probe student thinking and encourage student discussion to go even more in depth. Throughout the unit students are guided to constructive discussions when asked to explain their reasoning.

At the appropriate time in the building of understanding there will be an increased focus in asking students to express regularity to find the desired formulas increasing their proficiency in the counting procedures.

Minnesota State Mathematics Benchmarks Addressed:
9.4.3.1
Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities

Minnesota Comprehensive Assessment Question(s):

21. A group of health care providers consists of 4 doctors, 3 dentists, and 5 nurses. How many combinations of 2 health care providers of different types are possible?

24. There are four performers in a school talent show. In how many ways can the performers be arranged by different order of appearance?

A. 12
B. 16
C. 24
D. 256

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DAY 1: Counting Procedures Pretest

Counting Procedures Pretest Name_______________________________

1. Suppose you own a small deli. You offer 2 types of meat (ham and turkey), 2 types of cheese (cheddar and Swiss), and 2 types of bread (white and wheat). Draw a tree diagram to show how many choices your customers have for a meat sandwich.

2. You are going to set up a stereo system by purchasing separate components. In your price range you find 5 different receivers, 8 different compact disc players, and 12 different speaker systems. If you want one of each of these components, how many different stereo systems are possible?

3. The standard configuration for a New York license plate is 3 digits followed by 3 letters. How many different license plates are possible if digits and letters can be repeated?

4. To keep cell phones secure, many suggest the user to enter a pass code. The typical pass code is four characters long and can contain both numbers and letters. How many four-character passwords are possible if the first character is a digit and the last three characters are letters?

5. Find the number of distinguishable arrangements of the letters in the word LETTER.

6. There are four performers in a school talent show. In how many ways can the performers be arranged by different order of appearance?

7. Auditions are being held for the school play. The available roles for this play are *Teacher*, *Librarian*, and *Coach*. How many ways can the roles be assigned if 9 people audition?

8. The student body of 100 students wants to elect a president, vice president, secretary, and treasurer. In how many ways can this be accomplished?

9. A group of health care providers consists of 4 doctors, 3 dentists, and 5 nurses. How many combinations of 2 health care providers of different types are possible?

10. There are 50 people at a party. They each shake hands with everyone else. How many handshakes were there?

DAY 1: Introduction: The Handshake Problem

LAUNCH:
Imagine you are in a room full of people for a party, much like this classroom, and everyone shakes hands, how many total handshakes are there? We probably should have some basic ground rules for this party: you do not shake hands with yourself and you only shake hands once with another individual. We will demonstrate with 2 people, then 3 people, and then 4 people.

EXPLORE:
Working in groups (3 or 4 students per group), continue to keep track of the number of handshakes for each group size. Try to figure out how many handshakes there would be if all 20 of you shook hands. Students are encouraged to write it out in a table, to try it themselves, and/or look for patterns.

Groups will find the number of handshakes for each group size based off of what they discovered. Any discrepancies will be resolved during this time.

SHARE:
Which group wants to share a pattern they noticed?
Are there any other patterns groups have found?

SUMMARIZE:
Eventually most of you saw that every time a new person joins the group, that person has to shake hands with everyone else in the group. So when the group goes from six people (15 handshakes) to seven people, the 7^{th} person has to shake hands with six people. The new total is 21. What if 100 people were at the party and shook hands. How many handshakes would there be? By the end of this unit we will explore different patterns or a different methods to find the total number of handshakes. This unit will focus on counting procedures that will allow us to make a connection between what we have already noticed as a pattern and an efficient method for finding the total number of handshakes for 100 people, 1000 people, or even more.

DAY 2: Tables and Tree Diagrams
Mr. Potato Head

Source: http://www.amiddleschoolsurvivalguide.com/search?updated-max=2011-11-15T15:23:00-08:00&max-results=7&start=21&by-date=false

LAUNCH:
In many real-life problems you want to count the number of possibilities. For instance, suppose you own a Mr. Potato Head. You have lost some parts over the years, so you now have 2 types of legs, 2 types of eyes, 2 types of ears, and 2 types of lips. How many choices do you have for creating a Mr. Potato Head?

Each student group (3-4 students per group) is given a set of Mr. Potato Head (each group set contains one body and four different sets of pairs; i.e. two sets of feet, two sets of eyes, two sets of ears, and two sets of lips, but this could easily be differentiated). Work with your group to create and record the possible Mr. Potato Heads based on the parts you have in your set.

EXPLORE:
Working in groups (3 or 4 students per group), students create as many Mr. Potato Heads as they can using one of each "part" in a given amount of time and keep a record of their creations. After the given amount of time to work with the Mr. Potato Head sets and recording creations, the class discusses strategies they used to record creations.

How did groups decide to record the possible Mr. Potato Heads?
What was frustrating or difficult about some strategies?

Move on to discussing and emphasizing the strategy of tree diagrams with the class (hopefully at least one group will have employed this strategy to allow for an easy transition).

Which group can display the tree diagram for their Mr. Potato Head set?
Was this faster than other methods used by this class?

SHARE:
What is an effective method for displaying the Mr. Potato Head creations?

SUMMARIZE:
Tree diagrams are useful for organizing and visualizing the different possible outcomes of a sequence of events.

Consider the relationship between the elements of your tree diagram and the total possible outcomes of our tree diagram. Tomorrow we will work to discover this relationship to find an even faster strategy for finding the total possible combinations for a full Mr. Potato Head set.

DAY 3: Fundamental Counting Principle

LAUNCH:
Consider having all the parts to a Mr. Potato Head set (show an image of a complete set). How large would this tree diagram be? In general, how can you find the total number of possibilities for a complete Mr. Potato Head set? Did anyone notice a relationship between your answer yesterday and the number of parts within your set?

EXPLORE:
Working in groups (3 or 4 students per group), students will complete exercises using their preferred strategy.

- You are ordering lunch. The only choice for an entrée is a hamburger. You need to choose chips or fries. The drink options are bottled water, lemonade, or apple juice. Find the number of possibilities for your lunch.
- You are planning activities for the weekend. On Saturday you can either go to the movies or to the mall. On Sunday you can choose to participate in one of the following sports events: basketball, soccer, flag football, or lacrosse. Find the number of possibilities for weekend activities.

What strategies were used to find all the possibilities?
How do you know that you have listed all possibilities?
Allow students to compare/critique the methods that are shared.

In general, how can you find the number of possibilities in situations like this?
Ideally, there will be a few students that will be able to articulate that the number of possibilities is the same as the product of the choices for each option (or category). So in general, you can multiply the number of choices that each option can occur. If students struggle to articulate the principle, consider referring back to the Mr. Potato Head example to help them generalize the rule.

Present additional examples for group and class discussion.

- The store at your school wants to stock sweatshirts that come in four sizes (small, medium, large, x-large) and in two colors (red and white). How many different types of sweatshirts will the store have to stock?
- The call letters for all radio stations in the United States start with either a E (east of the Mississippi river) or a W (west of the Mississippi River) followed by three other letters that can be repeated. How many different call letters are possible?
- Joe has 5 shirts, 6 trousers, 3 ties, and 4 sport coats. How many different arrangements can he wear?
- Police use photographs of various facial features to help witnesses identify suspects. One basic identification kit contains 195 hairlines, 99 eyes and eyebrows, 89 noses, 105 mouths, and 74 chins and cheeks. A witness can clearly remember the hairline and the eyes and eyebrows of a suspect. How many different faces can be produced with this information?

SHARE:
What do Mr. Potato Head tree diagrams and all the problems we did today have in common?

SUMMARIZE:
All these problems use the Fundamental Counting Principle that allows you to multiply the number of choices that each option can occur to find the total number of outcomes.

DAY 4: Fundamental Counting Principle
Minnesota License Plates

LAUNCH:
Suppose you are on a road trip, someone else is driving, and you are trying to pass away the hours by staring out the window. Now after a while, this is bound to become boring, so you devise a game to play with yourself to stay entertained. You start looking at the cars as they pass and more importantly, their license plates. Each one seems to be different, all just a jumbled collection of characters including letters and/or numbers. You think to yourself, as any good mathematician would, "I wonder how many ways there are to create a unique license plates?"

Let's first focus on what we are familiar with, Minnesota automobile license plates (show them the class an old license plate). How many different license plates are possible with this arrangement?

EXPLORE:
Working in groups (3 or 4 students per group), students will find how many different plates are possible with the Minnesota automobile license plate using 3 digits first followed by 3 letters.

**Inform students that for this activity we will disregard the digit and letter arrangements the state does not allow for ease of calculations.

Which group will share their solution and method for finding the total possible different Minnesota automobile license plates?
Did anyone think of it differently?

Eventually Minnesota ran out of new numbers for automobile license plates (show the newer Minnesota automobile license plate with the order reversed – 3 letters first followed by 3 digits). How many different license plates are possible with this arrangement?

How is this scenario different than the first?
How is this scenario similar to the first?

SHARE:
What mathematical principle are we using to find all the possible Minnesota license plates?

SUMMARIZE:
The Fundamental Counting Principle allowed us to find the total possible different Minnesota automobile license plates by multiplying the number of choices that each element can occur to find the total number of outcomes. What would be your recommendation to Minnesota for the next time it runs out of new license plate numbers? Consider this over the next few lessons.

DAY 5: Fundamental Counting Principle
Research State License Plates

LAUNCH:
Minnesota is probably the coolest state and is certainly unique, but do you think Minnesota's method for arranging digits and letters is unique? Today you will explore two different states and their license plate arrangements. Like yesterday, we will disregard the digit and letter arrangements the state does not allow for ease of calculations.

Come draw two states from this bucket (slips of paper will have state names typed on them) and then search for information about each state's method for arranging digits and letters to create license plates.

EXPLORE:
Working individually, students will need to record observations about the arrangements for each state and compute the total possible license plates for each state based off of their observations. Students will then share with a partner their discoveries.

Students will be asked to share information about the state they thought had the most interesting arrangement and share how many possible license plates were possible using the arrangements. Have as many students share as time allows.

Why do you suppose states have different arrangements?
Did anyone's state have multiple forms of arrangements?
Why do you suppose some states keep changing the arrangement?

SHARE:
What mathematical principle are we using to find all the possible license plates for any state?

SUMMARIZE:
The Fundamental Counting Principle allowed us to find the total possible different automobile license plates for any state by multiplying the number of choices that each element can occur to find the total number of outcomes.

Tomorrow you will be given a population range and be asked to design a license plate arrangement that will fit within the given range to show your understanding of how these totals are computed.

DAY 6: Fundamental Counting Principle
Design License Plate

LAUNCH:
Think back to when I asked what would be your recommendation to Minnesota for the next time it runs out of new license plate numbers? After yesterday you should have more ideas for this. Let us list some possibilities as a class. (If no one thinks of not allowing repetition of letters and digits possibly suggest this if it something appropriate for the group).

Now your "state's" department of transportation employs your group. A group representative will have to come forward to draw a range of needed license plates for your group's "state." Each member of your group will have to create an arrangement to meet the range for needed license plates. You should have 3-4 different designs (depending on group size). Keep in mind the possibilities we have learned the past few days.

Ranges –

10,000-50,000	200,000-300,000	500,000-900,000
5,000,000-10,000,000	25,000,000-35,000,000	40,000,000-50,000,000

EXPLORE:
Working in groups (3 or 4 students per group), students will work on setting an arrangement to meet their range for needed license plates. Each group member has to have a different arrangement.

Are groups finding different arrangements to meet their assigned range?
What are some strategies some groups are using to find different arrangements?
Are the arrangements your group members are creating the only possibilities?

Each group will pick one member to present an arrangement that fits the assigned range. Presentations occur tomorrow.

SHARE:
What mathematical principle are we using to meet the range of the needed license plates?

SUMMARIZE:
The Fundamental Counting Principle allowed us to find the total possible different automobile license plates by multiplying the number of choices that each element can occur to find the total number of outcomes.

DAY 7: Fundamental Counting Principle
Present License Plates

LAUNCH:
State governments decide upon the specific arrangement of letters and numbers that will be used for each state. It is important to choose wisely on the pattern of possible numbers and letters, since the possible number of license plates will determine the number of cars that can be registered at any one time. As a member of your "state's" department of transportation you are now going to go before the "state" government, our class, and explain how your specific arrangement will meet the range needed for your "state."

EXPLORE:
Each group will have to share the range they drew and the arrangement they created to meet the range for needed license plates. Each group will have to demonstrate how they know their arrangement in fact does guarantee that they will have enough, but not too many license plates.

Did this group's arrangement meet their range requirement?
Are the arrangements this group created the only possibilities?
What could this group have done differently to stay within their range?
Do you think a smaller or larger range was easier to work with on this task? Why?

SHARE:
What mathematical principle are we using to meet the range of the needed license plates?

SUMMARIZE:
The Fundamental Counting Principle allowed us to find the total possible different automobile license plates by multiplying the number of choices that each element can occur to find the total number of outcomes.

License plates are just one example of applying the Fundamental Counting Principle to real world events. Can you think of any other things in the real world that this concept applies? Consider this before class tomorrow,

DAY 8: Pass codes and IDs

LAUNCH:
For many cell phones, the owner can set a 4-digit pass code to lock the device. Would it take someone very long to unlock your device?

EXPLORE:
Working in groups (3 or 4 students per group), students will discuss and calculate various scenarios involving passwords, IDs, etc. Periodically students will be asked to share their thinking with the entire class.

Consider a 4-digit pass code for a cell phone –
How many digits could you choose from for the first number of the pass code?
How many digits could you choose from for the second number of the pass code? Assume that the numbers can be repeated.
How many different 4-digit pass codes are possible?
How long (in hours) would it take someone to try every possible code if it takes three seconds to enter each possible code?

Now consider your lunch IDs –
How many digits are used for your lunch IDs?
How many lunch IDs are possible for the Pequot Lakes School District?
What would the average class size for the Pequot Lakes School District have to be for the district to run out of enough lunch codes for students using its current arrangement?

SHARE:
What mathematical principle are we using to find all of these codes, IDs, etc.?

SUMMARIZE:
The Fundamental Counting Principle allows you to make these calculations. The process of choosing the four digits for the pass code or four digits for your lunch ID involves a sequence of events. You can multiply the number of choices that each element in a pass code or ID card can occur.

DAY 9: Passwords

How Can We Make Stronger Passwords?

Source: http://robertkaplinsky.com/work/how-can-we-make-stronger-passwords/

LAUNCH:
Are your passwords safe? One common type of password hacking (or cracking) is called "brute force" and involves using a computer to try every single possible combination of characters until it enters the correct one. For example, if a password was one character long and could only be an English lowercase letter, then there would only be 26 possible passwords. If a hacker tried each of them, he or she would be guaranteed to figure out the password by the 26th attempt. Where the math gets interesting is figuring out what password requirements make them more complex or more specifically, are more effective in increasing the total number of possible passwords a hacker has to try.

Potential password requirements include –

- May use lowercase characters
- May use uppercase characters
- May use digits
- May use symbols
- Minimum number of characters
- Maximum number of characters

EXPLORE:
The students start working in groups to agree on a certain set of password requirements and then calculate the total number of potential passwords based on those requirements. Students will be encouraged to play around with different password requirements to see how they affect the total number of potential passwords.

Each group shares their set of password requirements and calculated potential passwords. Students will discuss how the different password requirements affected the total number of passwords. Then as a whole class, students agree on a certain set of password requirements and then calculate the total number of potential passwords based on those requirements.

SHARE:
What mathematical principle are we using to find all of these potential passwords?

SUMMARIZE:
The Fundamental Counting Principle allows us to find the number of potential passwords and although we were able to find very large totals, a large total number of potential passwords does not imply that the passwords are secure. Given more time to explore, you would discover that the single most important factor in making a password more complex is the password's length.

DAY 10: Distinguishable Arrangements
Arranging Letters

LAUNCH:
Consider the letters FUN. As any good mathematician would, we want to know how many distinguishable arrangements are there of the letters in FUN.

EXPLORE:
Working in groups (3 or 4 students per group), students will determine the total number of distinguishable arrangements.

Who found the number of distinguishable arrangements?
How did you determine this answer?
Did anyone do it differently?

Factorial

Additional words with no repeating letters (CAR, JAM, LEG, HAND, etc.) will be given until the class becomes comfortable with factorial.

Next, find how many distinguishable arrangements are there for the letters in FUNN.

Who found the number of distinguishable arrangements?
How did you determine this answer?
How is this different from arranging the letters in FUN?

Recall from a previous lesson that $n!$ is read as "n factorial." In general, the number of arrangements of n distinct objects is $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$. Also recall that you have this feature on your calculators.

Students will now have to build understanding that in order to make it a distinguishable arrangement they will need to account for the two Ns. Again, students will share thinking and additional words with repeating letters will be given until the class becomes comfortable with find distinguishable arrangements.

Word List –

BANANA	MISSISSIPPI	MONOPOLY	ALGEBRA
MATHEMATICS	GOOGLE	ARRANGEMENTS	APPLE

What group would like to share how they found the distinguishable arrangements?
Is it simply $n!$ for these?
What makes these different than the first set of problems?

SHARE:
What allows us to compute the total possible arrangements of a list of distinguishable letters?
What did we need to do when some of the letters in our list were not distinguishable?

SUMMARIZE:
The Fundamental Counting Principle can be used to determine the number of arrangements of *n* objects. In general, the number of arrangements of *n* distinct objects is $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$.

If some of the objects are repeated, then some of the arrangements are not distinguishable. The number of distinguishable arrangements of *n* objects where one object is repeated q_1 time, another is repeated q_2 times, and so on is: $\frac{n!}{q_1! \cdot q_2! \cdot \ldots \cdot q_k}$

DAY 11: Permutations

Ice Cream Cones

Source: Matsurra, R. (2013). MCTM 2013 spring conference ice cream cones & bowls [conference handout]. Department of Mathematics, St. Olaf College, Northfield, MN.

LAUNCH:
Nearly everybody loves ice cream. What are the top three flavors for our class? We are going to discover how many different ice cream cones we can create with our top three flavors if it is important which scoop is on the top.

EXPLORE:
Working in groups (3 or 4 students per group), students will be asked to consider task involving ice cream cones. Questions/tasks will be revealed one at a time. Sufficient time will be provided for each group to work and share thinking as a class.

ICE CREAM CONES
In an ice cream cone, it is important which scoop is on top. Thus, a vanilla-strawberry-chocolate cone is different from a strawberry-chocolate-vanilla cone. Also, you may not duplicate a flavor in a given cone. For example, vanilla-vanilla-chocolate is not a valid cone.

Suppose an ice cream shop has our three favorite ice cream flavors of ice cream
____________________, ____________________, and ____________________.
How many different three-scoop cones can you make using each of these flavors exactly once?

Suppose you want a four-scoop cone with your next favorite ice cream flavor
____________________. How many different cones can you make?

How many different cones can you make from 5 scoops of different flavors? 6 scoops? 7 scoops? 10 scoops?

How many different cones can you make from n scoops of different flavors? Explain how you know your answer is correct.

Suppose an ice cream shop serves 24 different flavors of ice cream.
How many different three-scoop cones can you make at the ice cream shop?
Note: As before, you cannot use the same flavor twice on one cone.

How many different four-scoop cones can you make at the ice cream shop?
Suppose you can make 156 different two-scoop cones at a certain ice cream shop. How many different flavors does this shop offer?

Suppose you can make 2,730 different three-scoop cones at a certain ice cream shop. How many different flavors does this shop offer?

SHARE:
What is the rule – first as a verbal description, then as a formula – for determining the number of different r-scoop cones you can make? (Base your rule on the 24-flavor ice cream shop.) How does our rule relate to our method for solving the previous cone problems?

SUMMARIZE:
A permutation is an arrangement of objects in a specific order.

The number of permutations can be found by taking the total number of arrangements of the objects we want, *n*! and dividing that by the number of objects we do not want, (*n* – *r*)!.

The number of permutations of *r* objects taken from a group of *n* distinct objects is denoted by ${}_nP_r$ and is given by: ${}_nP_r = \frac{n!}{(n-r)!}$

Our calculators have the ability to evaluate permutations.
MATH>PRB> ${}_nP_r$

DAY 12: Combinations

Ice Cream Bowls

Source: Matsurra, R. (2013). MCTM 2013 spring conference ice cream cones & bowls [conference handout]. Department of Mathematics, St. Olaf College, Northfield, MN.

LAUNCH:
Ice cream cones are great and allow for some order in placing your ice cream, but sometimes you do not want to pick an order or you want to be able to alternate between flavors. Bowls of ice cream are a great solution to this problem.

EXPLORE:
Working in groups (3 or 4 students per group), students will be asked to consider tasks involving ice cream bowls. Questions/tasks will be revealed one at a time. Sufficient time will be provided for each group to work and share thinking as a class.

ICE CREAM BOWLS
In a bowl of ice cream, the arrangement of scoops does not matter. For example, Cydney cannot tell the difference between a chocolate-vanilla bowl and a vanilla-chocolate bowl. As with cones, you cannot use the same flavor twice in one bowl. So, no vanilla-vanilla bowl.

At a certain ice cream shop, you can make 6 different two-scoop bowls of ice cream. How many different two-scoop cones can you make?

Find the number of flavors offered at this shop.

At another ice cream shop, you can make 465 different two-scoop bowls of ice cream. How many different two-scoop cones can you make?

If you can make 220 different three-scoop bowls of ice cream, how many different three-scoop cones can you make?

If you can make 210 different four-scoop bowls of ice cream, how many different four-scoop cones can you make?

If you can make 3,024 different four-scoop cones of ice cream, how many different four-scoop bowls can you make?

If you can make 55,440 different five-scoop cones of ice cream, how many different five-scoop bowls can you make?

An ice cream shop offers 24 flavors. How many different five-scoop bowls can you make?

SHARE:
What is the rule – first as a verbal description, then as a formula – for determining the number of different r-scoop bowls you can make? (Base your rule on the 24-flavor ice cream shop.) How does our rule relate to our method for solving the previous bowl problems?

SUMMARIZE:
A combination is an arrangement of objects in no specific order.

The number of combinations can be found by taking the total number of arrangements of the objects we want, $n!$ and dividing that by the number of objects we do not want, $(n - r)!$ and the repetitions that occur, $r!$. In other words, create all the permutations and divide by all the redundancies.

The number of combinations of r objects taken from a group of n objects is denoted by ${}_nC_r$ and is given by: ${}_nC_r = \frac{n!}{(n-r)!\cdot r!}$

Our calculators have the ability to evaluate combinations.
MATH>PRB> ${}_nC_r$

DAY 13: Combinations vs. Permutations
Locker Combinations or Locker Permutations?

LAUNCH:
What was the difference between the ice cream cones and the ice cream bowls? Were the rules we created for each different? For the ice cream cones, the order of the ice cream did matter. This is called a permutation. For the ice cream bowls, order did not matter. This is called a combination. As displayed in the ice cream examples, the number of possible arrangements changed when order did matter compared to when order did not, making it important to correctly determine whether a situation is a permutation or combination.

EXPLORE:
Working with a partner, students will decide if each scenario is a permutation or a combination. After a certain amount of time, partners will have to find another set of partners to compare answers. During this time students should work to settle any disagreements of solutions with sound reasoning of what makes something a permutation and what makes something a combination.

Possible Scenarios –

- In the Ugliest Dog competition, a blue ribbon will be awarded to the ugliest dog, a red ribbon to the second ugliest dog, and a yellow ribbon to the third ugliest dog. If there are 8 dogs in the competition, how many different ways can the ribbons be awarded?
- Brandon wants to take 3 dogs with him on his morning walk. If Brandon owns 8 dogs, how many different groups of dogs can he choose for his walk?
- Suppose a password requires three distinct letters. Find the number of three letter codes, if the letters may not be repeated.
- The high school track has 8 lanes. In the 100-meter dash, there is a runner in each lane. Find the number of ways that 3 out of the 8 runners can finish first, second, and third.
- There are 110 people at a meeting. They each shake hands with everyone else. How many handshakes were there?

Ask students as they work –
How do you know it is a combination and not a permutation?
How do you know it is a permutation and not a combination?

SHARE:
What makes a combination different than a permutation?

SUMMARIZE:
In English we use the word "combination" loosely, without thinking if the order of things is important. In mathematics we use more accurate language:

- If the order does not matter, it is a combination.
- If the order does matter it is a permutation.

DAY 14: Combinations vs. Permutations
Scavenger Hunt

Source: Sarah's School of Math via Teachers Pay Teachers

LAUNCH:
By now you are all counting procedure experts. Today you and a partner will get up and get moving to put your expertise of combinations and permutations to the ultimate test – a scavenger hunt. Problems have been placed throughout the school building. The top half of the sheet is the answer to a different problem and the bottom half is the problem on which you should work. Please spread out to different starting points and return to class when you end up at the problem at which you started. Take a notebook and a calculator with you for your adventure. Good luck.

EXPLORE:
- Each page is printed on different color paper and put up around the school (not in order).
- Student pairs can start at any location.
- The top half of the sheet is the answer to a different problem and the bottom half is the problem on which they should work.
- After they calculate their answer, they find the page around the school that has the answer and go there for their next problem, then repeat.
- Students should write down the letter of the problem, their work and answer for each problem to turn in after a class discussion.
- Students should end up at the problem at which they started.

Partners will be asked to list their scavenger hunt letters in the order they were solved. The class will compare orders, noting that all orders should be the same just with different starting positions. Any discrepancies in answers will be discussed and students will help each other clarify any confusing problems that occurred during the scavenger hunt.

SHARE:
How do we know something is a permutation and how do we compute it?
How do we know something is a combination and how do we compute it?

SUMMARIZE:
A permutation is an arrangement of objects in a specific order.
- The number of permutations can be found by taking the total number of arrangements of the objects we want, $n!$ and dividing that by the number of objects we do not want, $(n - r)!$.
- The number of permutations of r objects taken from a group of n distinct objects is denoted by ${}_nP_r$ and is given by: ${}_nP_r = \frac{n!}{(n-r)!}$
- Our calculators have the ability to evaluate permutations.
MATH>PRB> ${}_nP_r$

A combination is an arrangement of objects in no specific order.

- The number of combinations can be found by taking the total number of arrangements of the objects we want, $n!$ and dividing that by the number of objects we do not want, $(n - r)!$ and the repetitions that occur, $r!$. In other words, create all the permutations and divide by all the redundancies.
- The number of combinations of r objects taken from a group of n objects is denoted by ${}_nC_r$ and is given by: ${}_nC_r = \frac{n!}{(n-r)!\cdot r!}$
- Our calculators have the ability to evaluate combinations.
  MATH>PRB> ${}_nC_r$

S

24,310

A medical researcher needs 6 people to test the effectiveness of an experimental drug. If 13 people have volunteered for the test, in how many ways can 6 people be selected? Decide if it is a combination or permutation, then find the answer.

Q

1716

Fifty people purchase raffle tickets. Three winning tickets are selected and each person wins $500. In how many ways can the prizes be awarded? Decide if it is a combination or permutation, then find the answer.

R

19600

In a race in which six automobiles are entered, In how many ways can the first 4 finishers come in? Decide if it is a combination or permutation, then find the answer.

T

360

A book club offers a choice of 8 books from a list of 40. In how many ways can a member make a selection? Decide if it is a combination or permutation, then find the answer.

V

76,904,685

9 comedy acts will perform over 2 evenings. Five of the acts will perform on the first evening and the order in which the acts are performed is very important. How many ways can the schedule for the first evening be made? Decide if it is a combination or permutation, then find the answer.

A

15,120

Using 15 flavors of ice cream, how many cones with three different flavors can you create if it is important to you which flavor goes on top, middle, or bottom? Decide if it is a combination or permutation, then find the answer.

C

2730

Baskin-Robbins offers 31 flavors of ice cream. One of their items is a bowl consisting of three scoops of ice cream, each a different flavor. How many bowls are possible? Decide if it is a combination or permutation, then find the answer.

W

4495

An election ballot asks voters to select three city commissioners from a group of six candidates. In how many ways can this be done? Decide if it is a combination or permutation, then find the answer.

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D

20

Of 12 possible books, you plan to take 4 with you on vacation. How many different collections of 4 books can you take? Decide if it is a combination or permutation, then find the answer.

F

495

You volunteer to help drive children at a charity event to the zoo, but you can fit only 8 of 17 children present in your van. How many different groups of children can you drive? Decide if it is a combination or permutation, then find the answer.

DAY 15: Powerball Fever

Source: Ana Zuniga ©2016
https://www.teacherspayteachers.com/Store/Ana-Zuniga

LAUNCH:
On Wednesday, January 13, 2016, the Powerball Jackpot reached an estimated $1.5 billion, the largest in the world to date. Today, we are going to crunch some numbers to learn more about this *Powerball Fever*!

EXPLORE:
Working in groups (3 or 4 students per group), students will discuss and calculate questions regarding Powerball Fever.

Powerball Background information –
On October 4, 2015 the format of how to play Powerball changed. There used to be 59 white balls to choose from and 35 red balls to choose from. This is how to play today: 5 white balls are drawn out of a drum with 69 balls numbered 1 to 69, and one red ball is chosen out of a drum with 26 red balls numbered 1 to 26.

- Why do you think this change occurred?
- What do you think happened to the total number of possible combinations for playing the winning numbers? Did it increase or decrease?
- Find the total number of combinations for winning the January 2016 jackpot (correctly choosing all five white balls and one red ball). Show your work.
- If you purchased a ticket on September 4, 2015, how many total combinations were possible then? Show all your work.
- By changing the way the game is played on October 4, 2015, the number of combinations in fact ________________________ (increased/decreased).
- In order to guarantee that you will for sure hit the $1.5 billion jackpot, you'd have to purchase how many tickets?
- Let's pretend you actually have that much money to spend. You purchase all of the possibilities and hit the jackpot. It costs $2 per ticket. How much money have you spent? You opt to get a lump sum of $983.5 million (this does not include having to pay state taxes).

SHARE:
What counting procedure allowed us to find all the possible outcomes?

Extension
How do permutations and combinations relate to the Fundamental Counting Principle?

SUMMARIZE:
Counting procedures allow us to final all the possible outcomes.

The Fundamental Counting Principle is a way to figure out the total number of ways different events occur.

In general, the number of arrangements of *n* distinct objects is $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$.

A permutation is an arrangement of objects in a specific order.

- The number of permutations can be found by taking the total number of arrangements of the objects we want, $n!$ and dividing that by the number of objects we do not want, $(n - r)!$.
- The number of permutations of *r* objects taken from a group of *n* distinct objects is denoted by ${}_nP_r$ and is given by: ${}_nP_r = \frac{n!}{(n-r)!}$
- Our calculators have the ability to evaluate permutations.
  MATH>PRB> ${}_nP_r$

A combination is an arrangement of objects in no specific order.

- The number of combinations can be found by taking the total number of arrangements of the objects we want, $n!$ and dividing that by the number of objects we do not want, $(n - r)!$ and the repetitions that occur, $r!$. In other words, create all the permutations and divide by all the redundancies.
- The number of combinations of *r* objects taken from a group of *n* objects is denoted by ${}_nC_r$ and is given by: ${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$
- Our calculators have the ability to evaluate combinations.
  MATH>PRB> ${}_nC_r$

Counting Procedures Post Test

Counting Procedures Post Test Name________________________________

1. Suppose you own a small ice cream shop. You offer 2 types of cones (cake and waffle) and 5 types of ice cream (vanilla, chocolate, strawberry, mint, and Superman). Draw a tree diagram to show how many choices do your customers have for a single scoop ice cream cone.

2. A pizza shop runs a special where you can buy a large pizza with one cheese, one vegetable, and one meat for $9.00. You have a choice of 7 cheeses, 11 vegetables, and 6 meats. Additionally, you have a choice of 3 crusts and 2 sauces. How many different variations of the pizza special are possible?

3. The standard configuration for a Pennsylvania license plate is 3 digits followed by 4 letters. How many different license plates are possible if digits and letters can be repeated?

4. To keep cell phones secure, many suggest the user to enter a pass code. The typical pass code is four characters long and can contain both numbers and letters. How many four-character passwords are possible if the first character is a letter and the last three characters are digits?

5. Find the number of distinguishable arrangements of the letters in the word ALABAMA.

6. There are six performers in a school talent show. In how many ways can the performers be arranged by different order of appearance?

7. There are five finalists in the Mr. Pequot Lakes pageant. In how many ways may the judges choose a winner and a first runner-up?

8. A group of 45 people are going to run a race. The top three runners earn gold, silver, and bronze medals. In how many ways can the top three places occur?

9. A group of health care providers consists of 5 doctors, 4 dentists, and 3 nurses. How many combinations of 3 health care providers of different types are possible?

10. There are 110 people at a meeting. They each shake hands with everyone else. How many handshakes were there?